MATH 212: Homework 4

Due Tuesday, October 30

Numbered problems are from *Numerical Analysis* by L. Ridgway Scott. Other problems are from *An Introduction to Numerical Analysis* by Süli and Mayers (this is just for proper credit, you shouldn't need to reference that book).

Problem 1. Chapter 8: 7,19,22 Chapter 9: 9,10

Problem 2. Suppose that A is Hermitian and positive definite with $x^*Ax \geq \alpha\|x\|_2^2$. In class, and in the book, it was proved that Gauss-Seidel iteration converges for systems $Ax = f$, adapt that argument to get a bound on the convergence rate in terms of α and $\|A\|_2$.

Problem 3. Consider the tridiagonal matrix

$$A = \begin{pmatrix} 2 & -1 & & \\ -1 & \ddots & \ddots & \\ & \ddots & & -1 \\ & & -1 & 2 \end{pmatrix}.$$

The eigenvectors of A are given by

$$q_j = [\sin(j\theta), \sin(2j\theta), \dots, \sin(nj\theta)]^T \quad \text{for} \quad 1 \leq j \leq n$$

where $\theta = \frac{\pi}{n+1}$. Use this to compute the eigenvalues of A. Using the result of the previous problem derive a bound on the convergence rate for Gauss-Seidel iteration. Give a bound on how many steps it takes to reduce the initial error by a factor of $1/2$, how does this compare to banded direct methods?